

$$f_4(x+h) = f(x) + hf'(x) + \frac{1}{2}h^2f''(x) + \frac{1}{6}h^3f'''(x) + \frac{1}{24}h^4f^{(4)}(x)$$

$$f_4(x-h) = f(x) - hf'(x) + \frac{1}{2}h^2f''(x) - \frac{1}{6}h^3f'''(x) + \frac{1}{24}h^4f^{(4)}(x)$$

$$= 2f(x) + h^2f''(x) + \frac{1}{12}h^4f^{(4)}(x)$$

$$2f(x) + f_4(x+h) + f_4(x-h) - 2f(x) = h^2f''(x) + \frac{1}{12}h^4f^{(4)}(x)$$

$$\frac{1}{h^2}(f_4(x+h) - 2f(x) + f_4(x-h)) = f''(x) + \frac{1}{12}h^2f^{(4)}(x)$$

$$f''(x) \approx \frac{1}{h^2}(f_4(x+h) - 2f(x) + f_4(x-h))$$

where the error is $\frac{1}{12}h^2f^{(4)}(x)$.